SAFE HANDS & IIT-ian's PACE

LEAP TEST# 13 (JEE) ANS KEY Dt. 04-01-2024

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	B	31	A	61	B
2	C	32	A	62	C
3	C	33	B	63	D
4	A	34	A	64	A
5	A	35	B	65	B
6	A	36	D	66	A
7	B	37	C	67	D
8	C	38	B	68	B
9	B	39	C	69	A
10	A	40	B	70	B
11	D	41	D	71	D
12	B	42	D	72	C
13	A	43	B	73	C
14	D	44	A	74	D
15	A	45	C	75	B
16	D	46	C	76	A
17	B	47	C	77	B
18	C	48	D	78	B
19	B	49	D	79	B
20	D	50	C	80	B
21	6	51	4	81	3
22	3	52	BONUS	82	6
23	-30	53	53	83	5
24	2	54	4	84	8
25	46	55	6	85	6

See Maths solutions on next page.....

SAFE HANDS & PACE

: ANSWER KEY :

61)	b	62)	c	63)	d	64)	a	77)	b	78)	b	79)	b	80)	b
65)	b	66)	a	67)	d	68)	b	81)	3	82)	6	83)	5	84)	8
69)	a	70)	b	71)	d	72)	c	85)	6						
73)	c	74)	d	75)	b	76)	a								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(b)**

$A \equiv (\alpha, 2\alpha + 3), BC = 1$ unit. Equation of BC is $y - 3 = 0$

Distance of A from BC is $p \Rightarrow |2\alpha + 3 - 3|$

Area of $\Delta ABC = \Delta = |\alpha|; 5 \leq \Delta < 6 \Rightarrow 5 \leq |\alpha| < 6$

62 **(c)**

Clearly, the equation of PQ in the new position is $x = 2$

63 **(d)**

The combined equation of bisectors of angles between the lines of the first pair is

$$\frac{x^2 - y^2}{2 - 1} = \frac{xy}{9}$$

As these equations are the same, the two pairs are equally inclined to each other

64 **(a)**

Let $x_1 = a, x_2 = ar$ and $x_3 = ar^2; y_1 = b, y_2 = br$ and $y_3 = br^2$. Now,

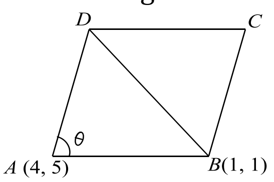
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{br - b}{ar - a} = \frac{b}{a}$$

And $\frac{y_3 - y_2}{x_3 - x_2} = \frac{br^2 - br}{ar^2 - ar} = \frac{b}{a}$

Therefore, slope of PQ is equal to slope of QR. Hence, points P, Q, R are collinear

65 **(b)**

From the figure



Area of rhombus $= 2 \times$ (area of ΔABD)

$= 2 \times \frac{1}{2} \times 5 \times 5 \sin\theta$

$= 25 \sin\theta$

Hence, maximum area is 25 (when $\sin\theta = 1$)

66 **(a)**

Acute angle between the lines $x^2 + 4xy + y^2 = 0$ is $\tan^{-}(2\sqrt{4-1})/(1+1) = \tan^{-1}\pi/3$. Angle bisector of $x^2 + 4xy + y^2 = 0$ are given by

$$\frac{x^2 - y^2}{1 - 1} = \frac{xy}{2}$$

$x^2 - y^2 = 0 \Rightarrow x = \pm y$

As $x + y = 0$ is perpendicular to $x - y = 4$, the given triangle is isosceles with vertical angle equal to $\pi/3$ and hence it is equilateral

67 **(d)**

Let (h, k) be the point on the locus. Then by the given conditions,

$(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$\Rightarrow 2h(a_1 - a_2) + 2k(b_1 - b_2) + a_2^2 + b_2^2 - a_1^2 - b_1^2 = 0$

$\Rightarrow h(a_1 - a_2) + k(b_1 - b_2) + \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$ (i)

Also, since (h, k) lies on the given locus, therefore

$(a_1 - a_2)h + (b_1 - b_2)k + c = 0$ (ii)

Comparing Eqs. (i) and (ii), we get

$$c = \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$$

68 **(b)**

If the line cuts off the axes at A and B, then area of triangle is $\frac{1}{2} \times OA \times OB = T$

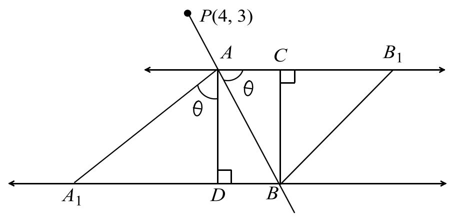
$\Rightarrow \frac{1}{2} \times a \times OB = T \Rightarrow OB = \frac{2T}{a}$

Hence, the equation of line is

$$\frac{x}{-a} + \frac{y}{2T/a} = 1$$

$\Rightarrow 2Tx - a^2y + 2aT = 0$

69 **(a)**



The given lines (L_1 and L_2) are parallel and distance between them (BC or AD) is $(15 - 5)/5 = 2$ units. Let $\angle BCA = \theta \Rightarrow AB = BC \operatorname{cosec}\theta$ and $AA_1 = AD \sec\theta = 2\sec\theta$. now area of parallelogram $AA_1\,BB_1$ is

$\Delta = AB \times AA_1 = 4\sec\theta\operatorname{cosec}\theta$

$$= \frac{8}{\sin 2\theta}$$

Clearly, Δ is least for $\theta = \pi/4$. Let slope AB be m

Then, $1 = \left|\frac{m + 3/4}{1 - \frac{3m}{4}}\right|$

$\Rightarrow 4m + 3 = \pm(4 - 3m) \Rightarrow m = 1/7$ or -7

Hence, the equation of 'L' is

$x - 7y + 17 = 0$

or $7x + y - 31 = 0$

70 **(b)**

Suppose we rotate the coordinate axes in the anticlockwise direction through an angle α. The equation of the line L with respect to old axes is

$\frac{x}{a}+\frac{y}{b}=1$

In this equation replacing x by $x\cos\alpha - y\sin\alpha$ and y by $x\sin\alpha + y\cos\alpha$, the equation of the line with respect to new axes is

$$\frac{x\cos\alpha - y\sin\alpha}{a}+\frac{x\sin\alpha + y\cos\alpha}{b}=1$$

$$\Rightarrow x\left(\frac{\cos\alpha}{a}+\frac{\sin\alpha}{b}\right)+y\left(\frac{\cos\alpha}{b}-\frac{\sin\alpha}{a}\right)=1 \quad \text{(i)}$$

The intercepts made by (i) on the coordinates axes are given as p and q

Therefore, $\frac{1}{p}=\frac{\cos\alpha}{a}+\frac{\sin\alpha}{b}$

and $\frac{1}{q}=\frac{\cos\alpha}{b}-\frac{\sin\alpha}{a}$

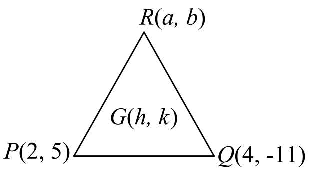
Squaring and adding, we get

$$\frac{1}{p^2}+\frac{1}{q^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

71 **(d)**

$R(x,y)$ lies on $9x+7y+4=0$

Hence, $R\left(a\frac{-(4+9a)}{7}\right), a\in R$



$h=\left(\frac{2+4+a}{3}\right)=\frac{6+a}{3}$ (i)

$$k=\frac{5-11-\frac{(4+9a)}{7}}{3}$$

$=\frac{-46-9a}{7\times3}$ (ii)

From (i) and (ii), we get

$$3h-6=\frac{-(21k-46)}{9}$$

$\Rightarrow\ 27h+21k-54+46=0$

Hence, the locus is $9x+7y-8/3=0$. This line is parallel to N

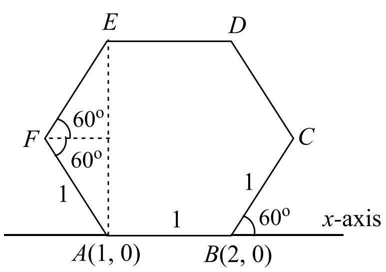
72 **(c)**

For any point $P(x,y)$ that is equidistant from given line, we have

$x+y-\sqrt{2}=-(x+y-2\sqrt{2})$

$\Rightarrow\ 2x+2y-3\sqrt{2}=0$

73 **(c)**



$$\Rightarrow C\equiv(2+1\times\cos 60°, 1\times\sin 60°)=\left(\frac{5}{2},\frac{\sqrt{3}}{2}\right)$$

$E\equiv(1, 1\times\sin 60°+1\times\sin 60°)=(1,\sqrt{3})$

Therefore, the equation of CE is

$$y-\sqrt{3}=\frac{\sqrt{3}-\frac{\sqrt{3}}{2}}{1-\frac{5}{2}}(x-1)$$

74 **(d)**

Clearly, A will remain as (0, 0); 'f_1' will make B as (0,4); 'f_2' will make it (12, 4) and 'f_3' will make it (4,8); 'f_1' will make 'C' as (2, 4); 'f_2' will make it (14,4); 'f_3' will make it (5,9). Finally 'f_1' will make 'D' as (2,0) 'f_2' will make ixt (2, 0) 'f_3' will make it (1, 1). So we finally get $A\equiv(0,0), B\equiv(4,8), C\equiv(5,9), D\equiv(1,1)$. Hence,

$$m_{AB}=\frac{8}{4}, m_{BC}=\frac{9-8}{5-4}=1, m_{CD}=\frac{9-1}{5-1}$$

$$=\frac{8}{4}, m_{AD}=1, m_{AC}=\frac{9}{5}, m_{BD}$$

$$=\frac{8-1}{4-1}=7/3$$

Hence, the final figure will be a parallelogram

75 **(b)**

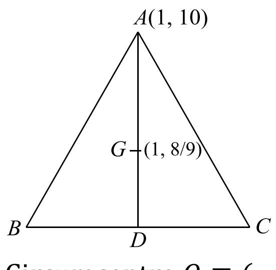
Let the coordinates of the third vertex be $(2a,t)$. Now, $AC=BC$. Hence,

$$t=\sqrt{4a^2+(a-t)^2}\ \Rightarrow t=\frac{5a}{2}$$

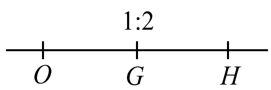
So the coordinates of third vertex C are $(2a, 5a/2)$. Therefore, area of the triangle is

$$\pm\frac{1}{2}\begin{vmatrix}2a & \frac{5a}{2} & 1\\ 2a & 0 & 1\\ 0 & a & 1\end{vmatrix}=\begin{vmatrix}a & \frac{5a}{2} & 1\\ 0 & -\frac{5a}{2} & 0\\ 0 & a & 1\end{vmatrix}=\frac{5a^2}{2}\text{ sq. units}$$

76 **(a)**



Circumcentre $O\equiv(-1/3, 2/3)$ and orthocenter $H\equiv(11/3,\ 4/3)$



Therefore, the coordinates of G are (1, 8/9), now, the point A is (1, 10) as G is (1, 8/9). Hence, $AD:DG=3:1$

$$\therefore\ D_x=\frac{3-1}{2}=1,\qquad D_y=\frac{\frac{8}{3}-10}{2}=-\frac{11}{3}$$

Hence, the coordinates of the midpoint of BC are $(1,-11/3)$

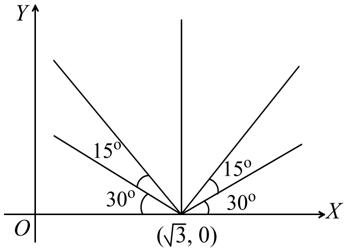
77 **(b)**

The given equation of pair of straight lines can be rewritten as $(\sqrt{3}y - x + \sqrt{3})(\sqrt{3}y + x - \sqrt{3}) = 0$

Their separate equations are $\sqrt{3}y - x + \sqrt{3} = 0$ and $\sqrt{3}y + x - \sqrt{3} = 0$

or $y = \frac{1}{\sqrt{3}}x - 1$ and $y = -\frac{1}{\sqrt{3}}x + 1$

or $y = (\tan 30°)x - 1$ and $y = (\tan 150°)x + 1$



After rotation through an angle of 15°, the lines are $(y - 0) = \tan 45° (x - \sqrt{3})$ and $(y - 0) - \tan 135° (x - \sqrt{3})$ or $y = x - \sqrt{3}$ and $y = -x + \sqrt{3}$

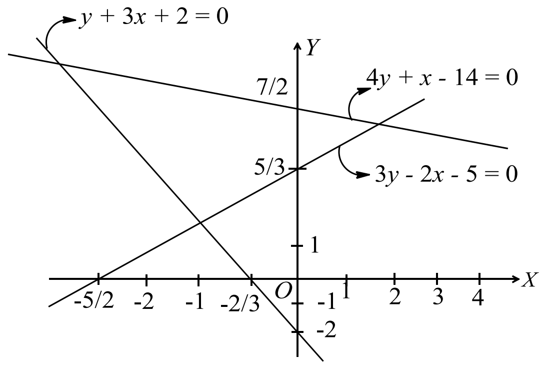
Their combined equation is

$(y - x + \sqrt{3})(y + x - \sqrt{3}) = 0$ or $y^2 - x^2 + 2\sqrt{3}x - 3 = 0$

Matrix Match Type

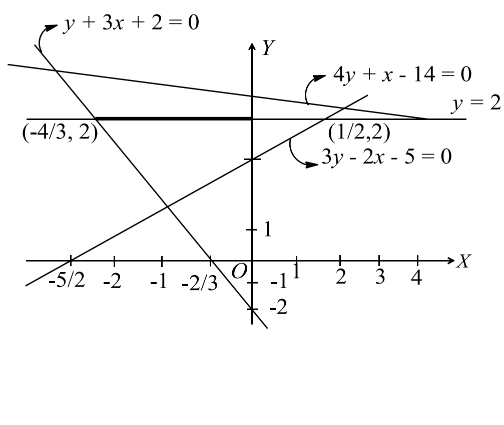
78 **(b)**

1.

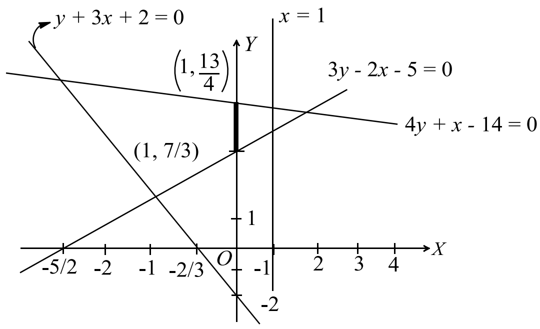


2. Clearly, point $(\alpha, 0)$ lies on the x-axis, which is not intersecting any side of triangle, hence no such α exists

3.



4.



Linked Comprehension Type

79 **(b)**

From the given equations we have,

$$\frac{1 - \cos\theta}{\sin\theta} = \frac{a - x}{y}$$

and $\frac{1 + \cos\theta}{\sin\theta} = \frac{a + x}{y}$

On multiplying, we get

$$\frac{1 - \cos^2\theta}{\sin^2\theta} = \frac{a^2 - x^2}{y^2}$$

$\Rightarrow x^2 + y^2 = a^2$

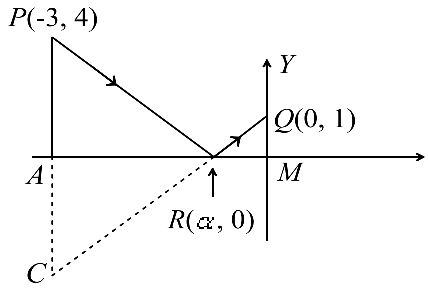
80 **(b)**

Equation of line through $(a, 0)$ be $\frac{x}{a} + \frac{y}{b} = 1$, where b is a parameter. This line meets y-axis at $(0, b)$ and if (h, k) denotes the mid point of the intercept of the line between the coordinate axes, then $h = \frac{a}{2}, k = \frac{b}{2}$ and then locus of (h, k) is $x = \frac{a}{2}$, which clearly does not intersect y-axis

Integer Answer Type

81 **(3)**

For $PR = RQ$ to be minimum it should be the path of light



$\therefore \angle PRA = \angle QRM$

From similar ΔPAR and $\Delta\ QMR$

$$\frac{AR}{RM} = \frac{PA}{QM}$$

$$\Rightarrow \frac{\alpha + 3}{0 - \alpha} = \frac{4}{1} \Rightarrow \alpha = -\frac{3}{5}$$

82 **(6)**

Let $x = r\cos\theta\,; y = r\sin\theta$

$\Rightarrow 2r\cos\theta + 3r\sin\theta = 6$

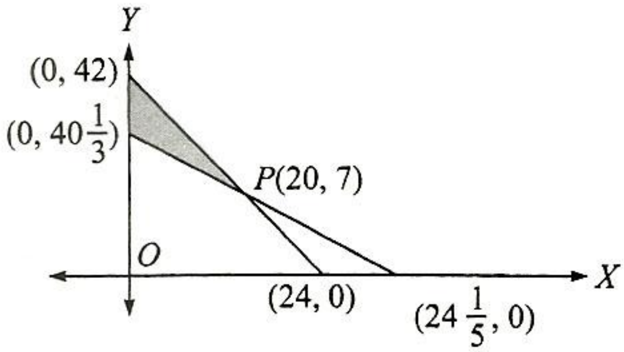
$\Rightarrow r = \frac{6}{2\cos\theta + 3\sin\theta}$; and $r = \sqrt{x^2 + y^2}$

for r to be minimum $2\cos\theta + 3\sin\theta$ must be maximum i.e., $\sqrt{13}$

$\therefore \ r_{\text{rim}} = \dfrac{6}{\sqrt{13}}$

83 **(5)**

The given lines $7x + 4y = 168$ and $5x + 3y = 121$ intersect $P(20, 7)$



$\therefore$ Area of shared region

$A = \dfrac{1}{2}\left(42 - 40\dfrac{1}{3}\right) 20$

$= \frac{1}{2}\left(\frac{5}{3}\right) 20 = \frac{50}{3}$ (square units)

84 **(8)**

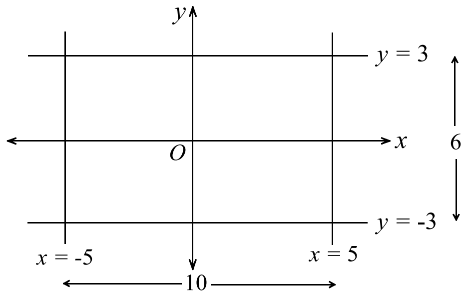
We know that the area of the triangle formed by joining the mid points of any triangle is one fourth of that triangle. Therefore required area is 8

85 **(6)**

$x^2y^2 - 9x^2 - 25y^2 + 225 = 0$

$\Rightarrow x^2(y^2 - 9) - 25(y^2 - 9) = 0$

$\Rightarrow (y^2 - 9)(x^2 - 25) = 0$



$\therefore$ Area $A = 10 \times 6 = 60$ sq. units